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ESTIMANDO A MÉDIA E A VARIÂNCIA
ESTIMANDO EL PROMEDIO Y VARIANZA
ESTIMATING THE MEAN AND VARIANCE

BMC Medical Research Methodology

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Estimating the mean and variance from the median, range, and the size of a sample

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ESTIMANDO A MÉDIA E A VARIÂNCIA
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ESTIMATING THE MEAN AND VARIANCE

m = median
a = the smallest value (minimum)
b = the largest value (maximum)
n = the size of the sample.

Mean: $$\bar{x} \approx \frac{a + 2m + b}{4} + \frac{a - 2m + b}{4n}$$

Standard deviation: $$s \approx \frac{b - a}{4}$$

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m = median = 49.7
a = minimum = 47.2
b = maximum = 52.0
n = the size of the sample = 20

50.1	49.6	49.1	48.1	51.7
49.8	50.6	51.8	48.8	49.9
52.0	48.7	47.2	50.4	51.0
50.7	49.3	49.5	49.5	48.5

Mean: $\bar{x} \approx \frac{a+2m+b}{4} + \frac{a-2m+b}{4n} = 49.6475$ (true mean = 49.815)

Desvio padrão: $s \approx \frac{b-a}{4} = 1.2$ (true sd = 1.263)

ESTIMANDO A MÉDIA E A VARIÂNCIA
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Wan *et al. BMC Medical Research Methodology* 2014, **14**:135
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RESEARCH ARTICLE Open Access

Estimating the sample mean and standard deviation from the sample size, median, range and/or interquartile range

Xiang Wan[1†], Wenqian Wang[2†], Jiming Liu[1] and Tiejun Tong[3*]

Abstract

Background: In systematic reviews and meta-analysis, researchers often pool the results of the sample mean and standard deviation from a set of similar clinical trials. A number of the trials, however, reported the study using the median, the minimum and maximum values, and/or the first and third quartiles. Hence, in order to combine results, one may have to estimate the sample mean and standard deviation for such trials.

Methods: In this paper, we propose to improve the existing literature in several directions. First, we show that the sample standard deviation estimation in Hozo et al.'s method (BMC Med Res Methodol 5:13, 2005) has some serious limitations and is always less satisfactory in practice. Inspired by this, we propose a new estimation method by

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m = median
a = the smallest value (minimum)
b = the largest value (maximum)
n = the size of the sample.

Standard deviation: $$s \approx \frac{b-a}{2\Phi^{-1}\left(\frac{n-0.375}{n+0.25}\right)}$$

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m = median = 49.7
a = minimum = 47.2
b = maximum = 52.0
n = the size of the sample = 20

50.1	49.6	49.1	48.1	51.7
49.8	50.6	51.8	48.8	49.9
52.0	48.7	47.2	50.4	51.0
50.7	49.3	49.5	49.5	48.5

Standard deviation: $$s \approx \frac{b-a}{2\Phi^{-1}\left(\frac{n-0.375}{n+0.25}\right)} = 1.2846$$ (true sd = 1.263)

```
> (b-a)/(2*qnorm((n-0.375)/(n+0.25)))
[1] 1.28463
```

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m = median
q_1 = the first quartile
q_3 = the third quartile
n = the size of the sample.

Mean: $$\bar{x} \approx \frac{q_1 + m + q_3}{3}$$

Standard deviation: $$s \approx \frac{q_3 - q_1}{2\Phi^{-1}\left(\frac{0.75n - 0.125}{n + 0.25}\right)}$$

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m = median = 49.7
q_1 = the first quartile = 49.025
q_3 = the third quartile = 50.625
n = the size of the sample = 20

50.1	49.6	49.1	48.1	51.7
49.8	50.6	51.8	48.8	49.9
52.0	48.7	47.2	50.4	51.0
50.7	49.3	49.5	49.5	48.5

Mean: $\bar{x} \approx \frac{q_1 + m + q_3}{3} = 49.783$ (true mean = 49.815)

Standard deviation: $s \approx \frac{q_3 - q_1}{2\Phi^{-1}\left(\frac{0.75n - 0.125}{n + 0.25}\right)} = 1.2765$ (true sd = 1.263)

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ESTIMATING THE MEAN AND VARIANCE

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- Luo, D., Wan, X., Liu, J., & Tong, T. (2018). Optimally estimating the sample mean from the sample size, median, mid-range, and/or mid-quartile range. *Statistical Methods in Medical Research*, *27*(6), 1785-1805.
  - Excel spread sheet is freely available at www.math.hkbu.edu.hk/~tongt/papers/optimalmean.xlsx
- Wan, X., Wang, W., Liu, J., & Tong, T. (2014). Estimating the sample mean and standard deviation from the sample size, median, range and/or interquartile range. *BMC Medical Research Methodology*, *14*(1), 135.

DIFERENÇA ENTRE MÉDIAS
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DIFFERENCE BETWEEN MEANS

Consider the data reported in Normand (1999) on the length of the hospital stay of stroke patients under specialized care and under conventional/routine non-specialist care:

```
> library(metafor)
> dat.normand1999
  study              source n1i m1i sd1i n2i m2i sd2i
1     1           Edinburgh 155  55   47 156  75   64
2     2      Orpington-Mild  31  27    7  32  29    4
3     3 Orpington-Moderate  75  64   17  71 119   29
4     4    Orpington-Severe  18  66   20  18 137   48
5     5       Montreal-Home   8  14    8  13  18   11
6     6  Montreal-Transfer  57  19    7  52  18    4
7     7           Newcastle  34  52   45  33  41   34
8     8                Umea 110  21   16 183  31   27
9     9             Uppsala  60  30   27  52  23   20
>
```

n1i : sample size, group 1
m1i : mean length of stay (in days), group 1
sd1i : standard deviation, group2
n2i : sample size, group 2
m2i : mean length of stay (in days), group 2
sd2i : standard deviation, group2

DIFERENÇA ENTRE MÉDIAS
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"MD": The raw mean difference is equal to m1i - m2i
"SMD": The standardized mean dierence is equal to (m1i - m2i)/spi, where spi is the pooled standard deviation of the two groups

```
> meta <- escalc(measure="MD", m1i=m1i, sd1i=sd1i, n1i=n1i,
                               m2i=m2i, sd2i=sd2i, n2i=n2i, data=dat.normand1999)
> meta
  study              source n1i m1i sd1i n2i m2i sd2i       yi       vi
1     1           Edinburgh 155  55   47 156  75   64 -20.0000  40.5080
2     2     Orpington-Mild  31  27    7  32  29    4  -2.0000   2.0806
3     3 Orpington-Moderate  75  64   17  71 119   29 -55.0000  15.6984
4     4   Orpington-Severe  18  66   20  18 137   48 -71.0000 150.2222
5     5      Montreal-Home   8  14    8  13  18   11  -4.0000  17.3077
6     6  Montreal-Transfer  57  19    7  52  18    4   1.0000   1.1673
7     7           Newcastle  34  52   45  33  41   34  11.0000  94.5891
8     8               Umea 110  21   16 183  31   27 -10.0000   6.3109
9     9             Uppsala  60  30   27  52  23   20   7.0000  19.8423
>
```



DIFERENÇA ENTRE MÉDIAS
DIFERENCIAS ENTRE PROMEDIOS
DIFFERENCE BETWEEN MEANS

```
> model1 <- rma(yi, vi, data=meta)
> model1

Random-Effects Model (k = 9; tau^2 estimator: REML)

tau^2 (estimated amount of total heterogeneity): 684.6462 (SE = 359.7541)
tau (square root of estimated tau^2 value):      26.1657
I^2 (total heterogeneity / total variability):   98.97%
H^2 (total variability / sampling variability):  97.21

Test for Heterogeneity:
Q(df = 8) = 238.9158, p-val < .0001

Model Results:

estimate      se     zval    pval     ci.lb   ci.ub
-15.1060  8.9466  -1.6885  0.0913  -32.6409  2.4289  .

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

>
```

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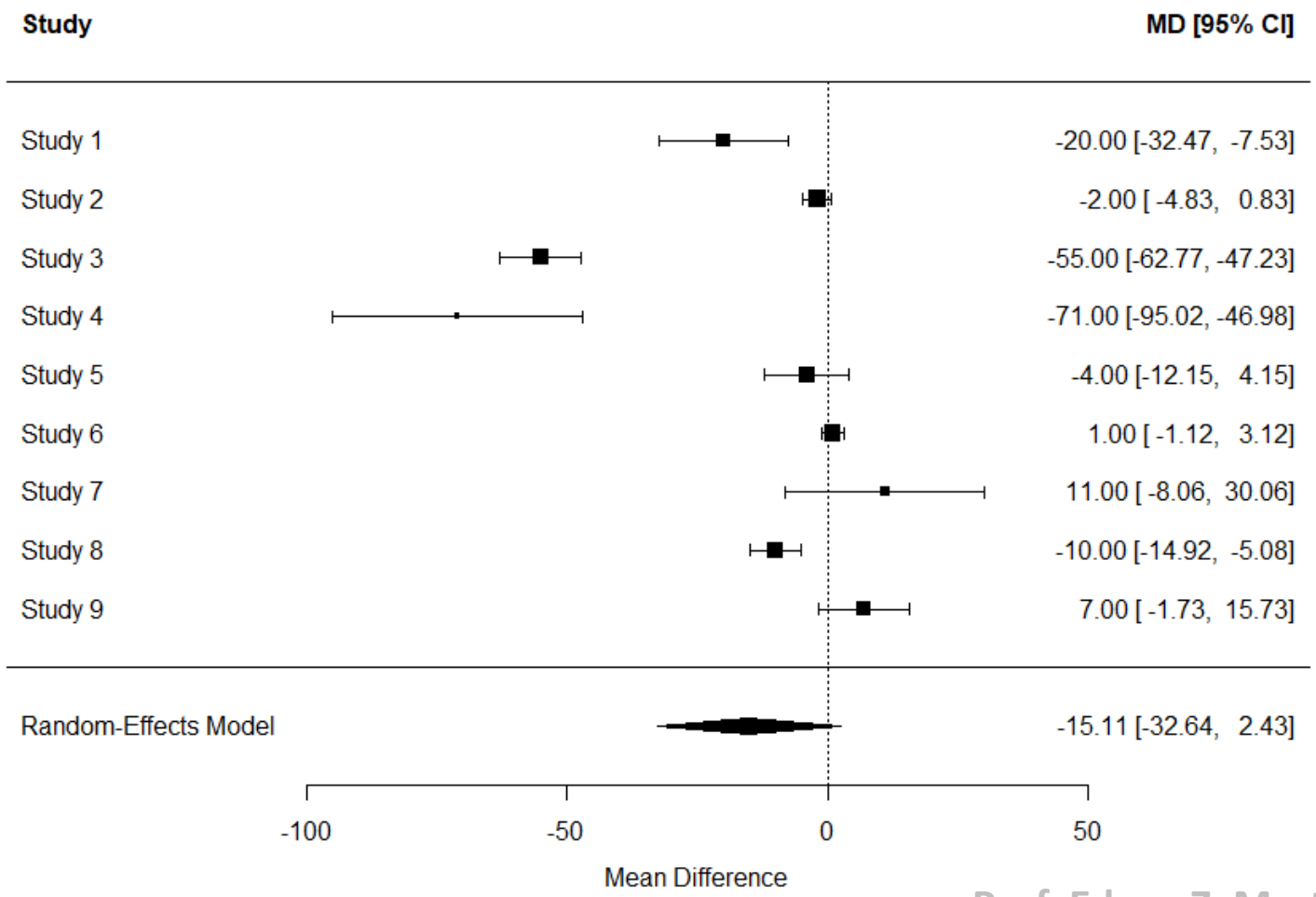
DIFFERENCE BETWEEN MEANS

```
> forest(model1)
```



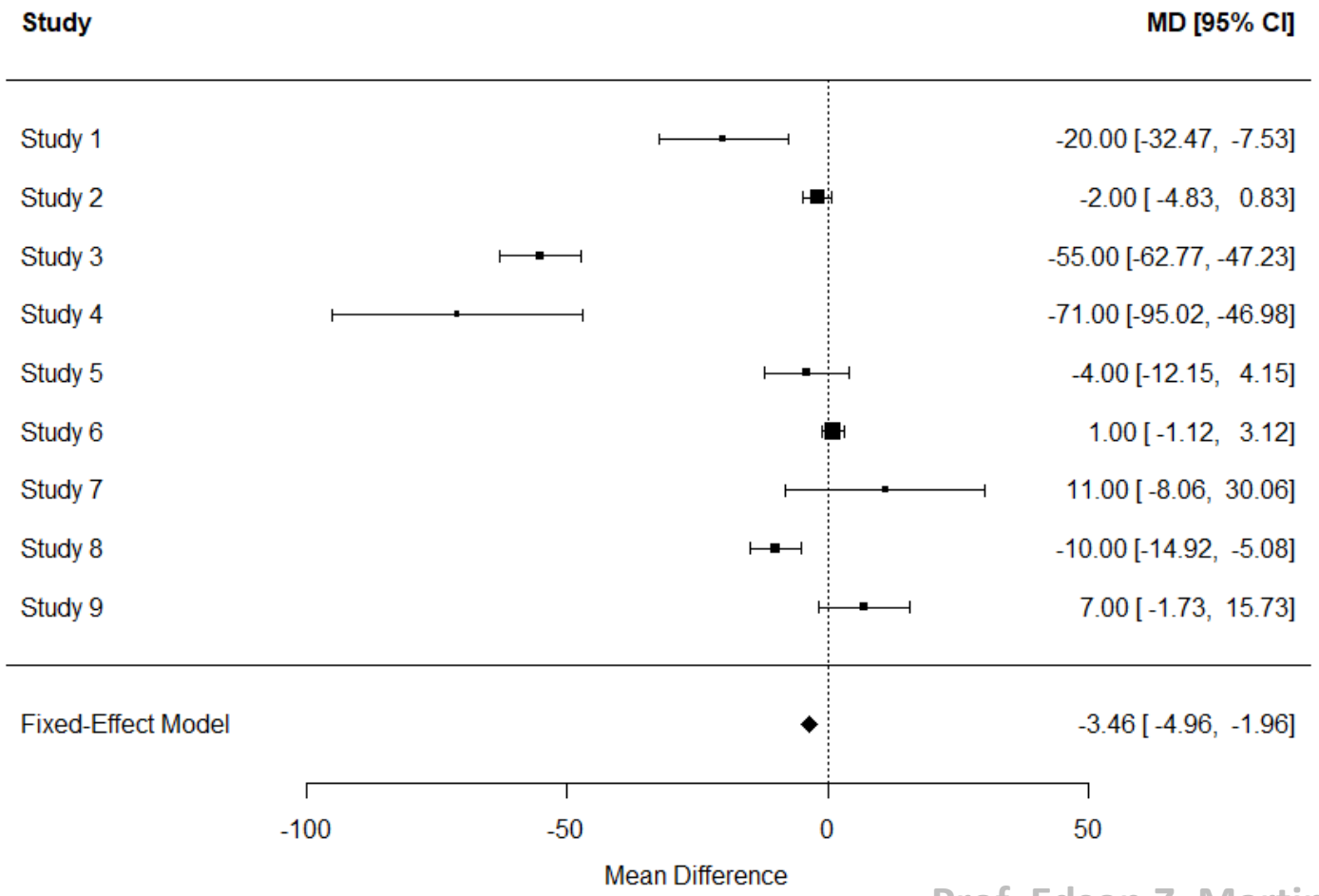


DIFERENÇA ENTRE MÉDIAS
DIFERENCIAS ENTRE PROMEDIOS
DIFFERENCE BETWEEN MEANS

```
> model2 <- rma(yi, vi, data=meta, method="FE")
> model2

Fixed-Effects Model (k = 9)

I^2 (total heterogeneity / total variability):   96.65%
H^2 (total variability / sampling variability):  29.86

Test for Heterogeneity:
Q(df = 8) = 238.9158, p-val < .0001

Model Results:

estimate      se     zval    pval    ci.lb    ci.ub
 -3.4636  0.7648  -4.5286  <.0001  -4.9626  -1.9646  ***

---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

DIFERENÇA ENTRE MÉDIAS
DIFERENCIAS ENTRE PROMEDIOS
DIFFERENCE BETWEEN MEANS

```
> forest(model2)
```





DIFERENÇA ENTRE MÉDIAS
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DIFFERENCE BETWEEN MEANS

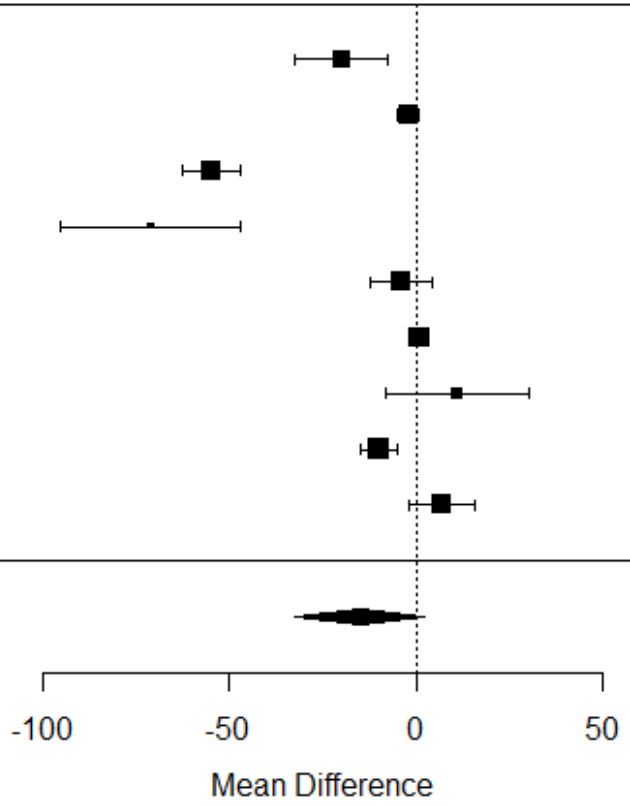
```
model3 <- rma(yi, vi, data=meta, slab=source)
forest(model3,xlim=c(-350,160), mlab="", header=FALSE,
       ilab=cbind(dat.normand1999$m1i,
                  dat.normand1999$sd1i,
                  dat.normand1999$m2i,
                  dat.normand1999$sd2i),
       ilab.xpos=c(-220,-190,-150,-120))

text(c(-220,-190,-150,-120), 11,   c("Mean", "SD", "Mean", "SD"), font=2)
text(c(-205,-135),           11.8, c("Treated", "Control"), font=2)
text(-350,                   11,   "Author",  pos=4, font=2)
text(160,                    11,   "Diff. [95% CI]", pos=2, font=2)

text(-350, -1, pos=4, cex=0.8, bquote(paste("RE Model for all studies (Q = ",
     .(formatC(model3$QE, digits=2, format="f")), ", df = ", .(model3$k - model3$p),
     ", p = ", .(formatC(model2$QEp, digits=2, format="f")), "; ", I^2, " = ",
     .(formatC(model3$I2, digits=1, format="f")), "%)")))
```

DIFERENÇA ENTRE MÉDIAS
DIFERENCIAS ENTRE PROMEDIOS
DIFFERENCE BETWEEN MEANS

Author	Treated Mean	Treated SD	Control Mean	Control SD	Diff. [95% CI]
Edinburgh	55	47	75	64	-20.00 [-32.47, -7.53]
Orpington-Mild	27	7	29	4	-2.00 [-4.83, 0.83]
Orpington-Moderate	64	17	119	29	-55.00 [-62.77, -47.23]
Orpington-Severe	66	20	137	48	-71.00 [-95.02, -46.98]
Montreal-Home	14	8	18	11	-4.00 [-12.15, 4.15]
Montreal-Transfer	19	7	18	4	1.00 [-1.12, 3.12]
Newcastle	52	45	41	34	11.00 [-8.06, 30.06]
Umea	21	16	31	27	-10.00 [-14.92, -5.08]
Uppsala	30	27	23	20	7.00 [-1.73, 15.73]
RE Model for all studies (Q = 238.92, df = 8, p = 0.00; I^2 = 99.0%)					-15.11 [-32.64, 2.43]



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